Exercise 9

(1) $P[W] = 6.763 \times 10^{-18} (\mathcal{E}[GeV])^2 (B[T])^2$ per e^-

for ultrarelativistic electrons

$I = 200\,mA$ is equivalent to $\frac{0.2}{1.6\times10^{-19}}$ e^- per sec.

Put in numbers

$P_{ring}[W] = 6.763\times10^{-18} \times 6^2 \times 0.8^2 \times \frac{0.2}{1.6\times10^{-19}} = 2600\,W.$ for the whole ring.

This only considers the bending magnets.

~~(2) The equation for radiation must be modified for photons~~

$$\cancel{P[W] = \frac{2}{3}\frac{e^4 c^3}{(m_p c^2)^4}(\mathcal{E}[GeV])^2 (B[T])^2}$$

$$\cancel{= 2.31\times10^{-11}(\mathcal{E}[GeV])(B[T])^2} \text{ per } e^-$$

~~Put in numbers~~

$$\cancel{P_{ring} = 2.31\times10^{-11} \times 7000^2 \times}$$

Willmott eqn. (3.25)

(2) $\omega_c = \frac{3}{2}\gamma^2 \frac{eB}{m_p}$ where $\gamma = \frac{\mathcal{E}}{\mathcal{E}_{p,rest}}$

$$\omega_c = \frac{3}{2} \times \left(\frac{7\times10^{12}}{0.938\times10^9}\right)^2 \times \frac{1.6\times10^{-19}\times 8.3}{1.67\times10^{-27}} = 6.64\times10^{16}\,sec^{-1}$$

$$\hbar\omega_c = 1.05\times10^{-34} \times 6.64\times10^{16}$$

$$= 6.97\times10^{-18}\,J$$

$$= 0.043\,keV$$

The characteristic photon energy is too low for a hard X-ray source.

(3) $401\, e^{-3/\tau[\mathrm{min}]} = 399$

$$-\frac{3}{\tau} = \ln \frac{399}{401}$$

$$\tau = -3/\ln\left(\frac{399}{401}\right) = 600\ \mathrm{min} = \underline{10\ \mathrm{hr}}$$

(4) $$\rho_{\mathrm{effective}} = \frac{E[J]}{ceB} = \frac{8\times10^{9}\times1.6\times10^{-19}}{3\times10^{8}\times1.6\times10^{-19}\times0.68} = 39.2\ \mathrm{m}$$

$$2\pi\rho_{\mathrm{effective}} = 246\ \mathrm{m}$$

$$\frac{246}{1436} = \underline{17\%}.$$

$$\hbar\omega_c[\mathrm{keV}] = 0.665\left(E[\mathrm{GeV}]\right)^2 B[T].$$

$$= 0.665\times8^2\times0.68$$

$$\times\ \underline{= 16\ \mathrm{keV}}.$$

(5) $$m\lambda_m = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

$$(m+1)\lambda_{m+1} = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

$$E_m = \frac{hc}{\lambda_m} = \frac{hc}{\lambda_m}$$

$$E_{m+1} = \frac{hc}{\lambda_{m+1}}$$

$$\Delta E = E_{m+1} - E_m = \frac{hc}{\lambda_{m+1}} - \frac{hc}{\lambda_m}$$

$$= hc\,(m+1-m)\,\frac{2\gamma^2}{\lambda_u}\left(1+\frac{K^2}{2}\right)^{-1}$$

$$= \frac{2hc\gamma^2}{\lambda_u}\left(1+\frac{K^2}{2}\right)^{-1}$$

(6) Radiation due to an undulator away from the axis is maximal if the condition

$$m\lambda_m(\theta) = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

is satisfied.

$m = 5$, $K = 1.6$ $\quad \gamma = \frac{E}{E_{rest}} = \frac{2.4 \times 10^9}{0.511 \times 10^6} = 4700$

$$5\lambda_5(0) = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{1.6^2}{2}\right) = \frac{\lambda_u}{2\gamma^2} \times 2.28 \qquad (1)$$

$$5\lambda_5(\theta) = \frac{\lambda_u}{2\gamma^2}\left(\underbrace{1 + \frac{1.6^2}{2}}_{2.28} + \gamma^2\theta^2\right) \qquad (2)$$

~~from (1):~~

$$\cancel{\lambda_u = \frac{5\lambda_5(0)}{2.28} \times 2\gamma^2}$$

$$\frac{\lambda_5(0)}{\lambda_5(\theta)} = \frac{hc/E_5(0)}{hc/E_5(\theta)} = \frac{E_5(\theta)}{E_5(0)} = \frac{8.968}{9.399} = 0.9541$$

$$(1)/(2) \longrightarrow \ = \frac{2.28}{2.28 + 4700^2\theta^2}$$

$$2.28 \times 0.9541 + 0.9541 \times 4700^2\theta^2 = 2.28$$

$$\theta = \frac{1}{4700}\sqrt{\frac{2.28 \times (1 - 0.9541)}{0.9541}} = 7.04 \times 10^{-5} \text{ rad}$$

Opening angle of a bending magnet is

$$\theta_{BM} = K/\gamma = \frac{1.6}{4700} = 3.4 \times 10^{-4} \text{ rad}$$

The opening angle of the undulator is an order of magnitude smaller.